The TIMS Laboratory Method

Math Trailblazers is a comprehensive mathematics program that incorporates many important scientific ideas. Scientific concepts often provide contexts for developing and practicing math concepts and skills. The tools and processes of science are integral to mathematical problem solving throughout the curriculum.

This tutor expands upon the *Math Trailblazers* connection with science. It outlines the Teaching Integrated Math and Science (TIMS) Project's view of science and describes the TIMS Laboratory Method, a version of the scientific method. This method forms a framework throughout the curriculum for students to explore science in much the way scientists work.

PART I The TIMS View of Science

Traditionally, school science has focused on the results of science. Students learn about parts of the body, types of rocks, the solar system, evolution, and so on. Knowing basic facts of science is seen as part of being educated, today more than ever. However, the facts of science, important and interesting as they are, do not alone comprise a comprehensive and balanced science curriculum.

The great educator and philosopher John Dewey expressed this idea almost 100 years ago. In 1910, he wrote:

> *At times, it seems as if the educational availability of science were breaking down because of its sheer mass. There is at once so much of science and so many sciences that educators oscillate, helpless, between arbitrary selection and teaching a little of everything.*
>
> *Visit schools where they have taken nature study conscientiously. This school moves with zealous bustle from leaves to flowers, from flowers to minerals, from minerals to stars, from stars to the raw materials of industry, thence back to leaves and stones.*
>
> *Thus,. . . science teaching has suffered because science has been so frequently presented just as so much ready-made knowledge, so much subject-matter of fact and law, rather than as the effective method of inquiry into any subject-matter.*

Outline

This tutor is organized as follows:

Surely if there is any knowledge which is of most worth it is knowledge of the ways by which anything is entitled to be called knowledge instead of being mere opinion or guess-work or dogma.

Such knowledge ... is not information, but a mode of intelligent practise, an habitual disposition of mind.

In 1996, the National Research Council (NRC) published the *National Science Education Standards* for K–12 science education. Among the many recommendations of the NRC document is a direction for decreased emphasis on teaching scientific facts and information, and increased emphasis on teaching for understanding of scientific concepts and developing abilities of inquiry. The NRC *Standards* state:

Emphasizing active science learning means shifting away from teachers presenting information and covering science topics. The perceived need to include all the topics, vocabulary, and information in textbooks is in direct conflict with the central goal of having students learn scientific knowledge with understanding.

If we were to describe the TIMS approach in the most concise way possible, we would choose two words, variable *and* experiment.

These points of view underlie the TIMS approach to science. If we were to describe the TIMS approach in the most concise way possible, we would choose two words, *variable* and *experiment.* The essence of modern science, as it is practiced by scientists, is to understand the relationships among variables. Out of the great sea of variables we have selected those that we feel are fundamental to the understanding of all areas of science, namely: length, area, volume, mass, and time. These variables might be considered the fundamental vocabulary of science. They are integral to the everyday work of biologists, chemists, physicists, astronomers, and earth scientists. The more that a child has explored these variables, the greater will be his or her command of scientific language and the more complete will be his or her ability to take up the adventure of science.

We have therefore made these variables the focus of experiments and activities in Grades K through 5 of *Math Trailblazers*. Explorations in kindergarten are conceptual in nature. As the curriculum progresses through the grades, students revisit the variables many times in increasingly more sophisticated ways. In Grade 5, students are able to move on to compound variables such as density and velocity, which involve two of the basic variables. For example, density involves both mass and volume while velocity involves both length and time. To understand these compound variables, it is important that students are first familiar with the more basic variables. Through repeated investigation of the variables in different contexts, fundamental science concepts and skills become generalized.

PART II Variables in Scientific Experiments

A **variable** is a quantity that may assume any one of a set of values. The variable is the heart and soul of science because the variable is to scientific investigation what the word is to language—its foundation and the basis of its structure. All experiments center around at least two variables, and the ability to measure these variables satisfactorily will determine the success or failure of the experiment.

Variables and Values

Variables fall into two broad categories: categorical and numerical. Would you say that the color of a person's hair is a categorical or a numerical variable? Would you say that the height of a person is a categorical or numerical variable? A **numerical variable** is one that may assume a numerical set of values. In contrast, a **categorical variable** is one which does not assume numerical values.

Color, then, which can take on values such as red, blue, or yellow, is a categorical variable. Other categorical variables are shape, kind of object, and type of material of which the object is made. In each case, you have the broad classification, the variable, and then the values the variable can assume.

The simplest kind of numerical variable is the number of objects in a set. For example, if we were studying the number of students who came to class each day, the variable would be "Number of Students," and the possible values of the variable would be 0 students, 1 student, 2 students, etc. A second category of numerical variable is those involving measurement. The basic measurement variables stressed in *Math Trailblazers* are length, area, volume, mass, and time. The values for these variables are what we measure during the course of an experiment. For example, if we are investigating the variable length, the values might be the number of meters, centimeters, millimeters, or other appropriate unit of length. When the variable is area, the values might be the number of square centimeters. Or if volume is the variable, the value might be the number of cubic centimeters, and so on. The variable is the broad classification; the **values** for the variable describe what we are counting or measuring for that variable.

One point in studying variables that will take repeated practice for your students to master is the regular use of units of measure. In science, we never deal with numbers without understanding what their units are. For example, say that we tell you that Mary dropped a ball from a height of 30. A question your students will learn to ask is, "Thirty what?" Was it 30 centimeters, 30 feet, or 30 miles? The name after the number 30 is what we call the unit of measure, and every variable has a set of them. $5 + 4 = 9$ can be meaningless unless we know 5 of what. 5 apples + 4 apples = 9 apples, but 5 apples + 4 pears does not equal either 9 apples or 9 pears. The sum is equal to 9 pieces of fruit. Invariably, children will give you the numerical value of the variable and leave off its unit. Learning to use units is merely a matter of discipline; that is, using them correctly so often that you feel uncomfortable when you either forget them or use them incorrectly. Developing this discipline in your students will help them later in their schooling as they examine more complicated scientific concepts.

Manipulated and Responding Variables in Controlled Experiments

In an experiment, a scientist tries to find a relationship between two variables. Where possible, the experimenter chooses ahead of time the values of one of the variables. This variable is called the **manipulated variable.** The values of the second variable are determined by the results of the experiment—something that the experimenter does not know ahead of time. We shall call the variable whose values result from the experiment, the **responding variable.**

For example, consider the following situation from a fourth-grade experiment, *The Bouncing Ball.* In this experiment, children study how high a ball bounces. They drop a ball from various heights and measure how high it bounces. As the experimenters, they decide what drop heights they want to use in the experiment, say 40 cm, 80 cm, and 120 cm. The drop height is the manipulated variable. But the height to which the ball bounces can only be determined after you drop the ball. It is not known ahead of time. So, the bounce height is the responding variable.

Or, say that you fill a plastic jar with different-colored blocks and that you have chosen red, yellow, and blue as the colors of your blocks. You then ask a child to reach in and pull out as many pieces as he or she can in one grab, sort the blocks by color, and count them. Color is the manipulated variable in this experiment. The number of each color that he or she pulls out is the responding variable. You choose the values of the colors that go in but you have no choice over the number of each color that comes out.

Beginning in the first grade of *Math Trailblazers*, the experimental variables are identified in all laboratory experiments. This is an essential part of conducting any scientific experiment. It is not until fourth grade, however, that we introduce the formal terms "manipulated" and "responding."

Note that in *Math Trailblazers*, we have elected to use the terms "manipulated" and "responding" to describe experimental variables. We have found manipulated and responding to be less abstract for children (and adults) and easier to understand than the more commonly used terms for variables, "independent" and "dependent."

Fixed Variables in Controlled Experiments

Experiments often have more than two variables. In an ideal experiment, more easily realized in the laboratory than in the real world, a scientist focuses on only two variables—the ones we have called manipulated and responding—and strives to hold all others constant. If too many variables change simultaneously in an experiment, obtaining meaningful results can be difficult or impossible.

In an ideal experiment, more easily realized in the laboratory than in the real world, a scientist focuses on only two variables—the ones we have called manipulated and responding—and strives to hold all others constant. If too many variables change simultaneously in an experiment, obtaining meaningful results can be difficult or impossible.

Consider a laboratory experiment done in second grade, *Rolling Along in Centimeters.* Students roll different kinds of cars down a ramp and use metersticks to measure the distance each car rolls. Here, the "type of car" is the manipulated variable and the distance each car rolls is the responding variable. The height of the ramp, the starting line, the floor onto which the cars roll, and the method for releasing the car are all held constant each time a car is rolled down the ramp. We refer to these as the **fixed variables** (or controlled variables) in the experiment. Children intuitively understand this as "keeping things fair" while the data are collected during the course of the experiment.

In many situations, some of the variables in an experiment are hidden; that is, they are not immediately obvious, although changing them will greatly alter the results of your experiment. Consider an experiment in which you collect data on the kinds of pets owned by each child in the classroom. The kind of pet is the manipulated variable and the number of each kind of pet is the responding variable. If you think about it, you can see that your results depend upon where the pet owners live—in a high rise or a single-family house, in the city or a farm community, and even in which country. Thus the variable, "location," can drastically change the results.

One of the reasons cancer research is so difficult is because there are so many variables which are either difficult or impossible to control (such as environmental factors, personality traits, multitudinous viruses) that it is hard to pinpoint the variables that might be the cause of cancer. It is the complexity of biological and sociological systems that makes doing repeatable experiments in these areas very difficult.

Nonetheless, children can learn how to deal with simple physical systems where three or four variables are present. They should learn how to recognize the different variables involved and understand the importance of controlling all but two—the manipulated and the responding variable.

The TIMS Laboratory Method

PART III

Throughout *Math Trailblazers*, children carry out quantitative investigations in which they explore the relationships between variables. Each investigation is carried out with the same general format, which we call the TIMS Laboratory Method. One may call this our version of the scientific method. There are four phases: beginning the investigation and drawing the picture, collecting and organizing the data, graphing the data, and analyzing the experimental results. We discuss these four phases as follows, illustrating our discussion with references to *The Bouncing Ball* experiment described above and to *Marshmallows and Containers,* a second-grade experiment in which children study the number of marshmallows that fit into different-sized jars.

Phase 1: Beginning the Investigation and Drawing the Picture

Most investigations begin with a question. The question does not have to be momentous, but it must be meaningful to children. If the question connects in some authentic way with their experience, children will need no flashy inducements to want to find the answer. This is illustrated in the following classroom anecdote involving *Marshmallows and Containers.*

> *As class begins, students sit together in groups of three. Each group has three different containers: a margarine tub, a 100 cc graduated cylinder, and a small paper cup. The teacher shows the class a bag of miniature marshmallows and asks, "Which container will hold the most marshmallows? Why do you think so?"*
>
> *The teacher encourages the groups to discuss their predictions and explanations and to record their ideas in their journals. When the groups report to the class, many groups think that the graduated cylinder will hold the most because it is the tallest. One boy explains that he thinks the cylinder holds the most because even if he could stretch the plastic in the bowl so it was tall like the cylinder, it still would not be as tall and would not hold as much. Other groups choose the tub because it is fatter than the other containers.*

The teacher then asks, "How could you find out which container holds the most marshmallows?" This question leads naturally to an experiment: The students will fill each of the containers with marshmallows, count them, and record the numbers in a data table.

Once a suitable question has been posed—and posing such questions is far from trivial—then variables related to the question must be identified. *Marshmallows and Containers* examines the kind of container and the number of marshmallows. The size of the marshmallows and the method for packing the marshmallows into the jars is held fixed. In *The Bouncing Ball,* students measure the drop heights and bounce heights of a ball, while holding fixed the type of ball, the floor onto which the ball is dropped, and the method for dropping the ball.

Through class discussion, the original question has been refined into a precise query about the relationship between two variables. These variables become defined well enough so that the children know how to gather information about them. Drawing a picture is an excellent way to summarize and communicate this beginning phase, and also to plan what is to come.

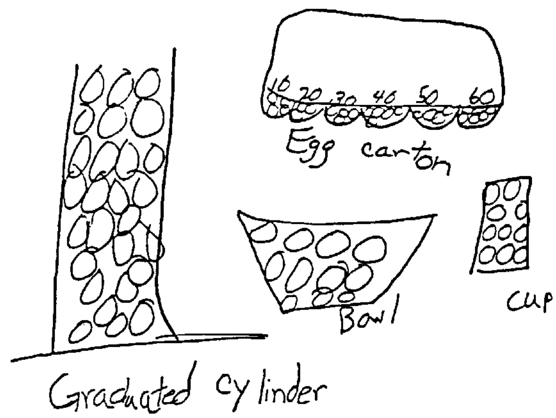

Figure 1: *Picture for* Marshmallows and Containers

Scientists often make sketches in experimental situations. In the TIMS Laboratory Method, drawing pictures helps children understand and organize what they are to do. A sketch gives the child time to think, to see relationships between variables, to place diverse relationships in a compact form, and to "explain" the experiment or problem to someone at a glance.

Scientists often make sketches in experimental situations. In the TIMS Laboratory Method, drawing pictures helps children understand and organize what they are to do. A sketch gives the child time to think, to see relationships between variables, to place diverse relationships in a compact form, and to "explain" the experiment or problem to someone at a glance. Pictures also help teachers assess whether students are ready to proceed. Figure 1 shows one student's picture for *Marshmallows and Containers.* The picture indicates the student's understanding of the experimental variables and the procedure. Note that the student is using an egg carton to make groups of 10 when counting marshmallows. Figure 2 shows a picture from *The Bouncing Ball*, indicating the manipulated and responding variables (the drop height and the bounce height) and the procedure for the experiment. The students who drew these pictures are ready to go on to the next step, gathering data.

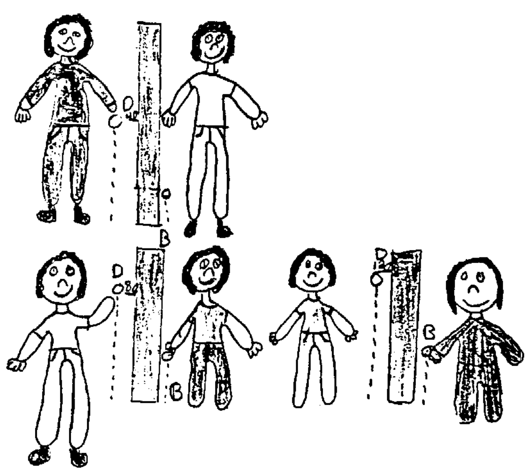

Figure 2: *Picture for* The Bouncing Ball

Phase 2: Collecting and Organizing the Data

In Phase 2 of the TIMS Laboratory Method, children gather the data and organize it in a table. To illustrate, we return to our classroom anecdote involving *Marshmallows and Containers:*

> *On the next day, the children use their pictures to review the experiment before beginning the data collection. Each group receives a two-column data table, fills in the column headings, and writes the names (or draws pictures) of the containers in the first column. The teacher emphasizes the need for accuracy in counting, and the children discuss various methods for grouping and counting the marshmallows. As the students begin to collect the data, the teacher circulates among the groups, coaching and assessing.*
>
> *After the children complete their data tables (Figure 3), the teacher leads a discussion in which the groups compare their results. This discussion centers on whether the groups' results are reasonable or not. Most groups report about 130 marshmallows in the graduated cylinder. Students agree that numbers close to 130, but not exactly 130, are acceptable. Based on this discussion, a group that had recorded only 110 marshmallows for the cylinder decides to refill the cylinder and count again.*

Container	N Number of Marshmallows
Graduated cylinder	133
cup	121
bowl	181

Figure 3: *Data table from* Marshmallows and Containers

Tennis Ball

D Drop Height in cm	B Bounce Height in cm			
	Trial 1	Trial 2	Trial 3	Average
40	27	22	21	22
80	50	40	43	43
120	72	62	68	68

Figure 4: *Data table from* The Bouncing Ball

Figure 3 shows a student's data table for *Marshmallows and Containers.* Figure 4 is a student's data table from *The Bouncing Ball*. The name of a variable, including units if appropriate, heads each column.

When creating data tables, scientists and mathematicians generally place the manipulated variable in the left column and the responding variable in the right column. Since the values for the manipulated variable are chosen by the experimenter before the experiment begins, the first column of the data table can be filled in prior to the experiment. Values of the responding variable can only be filled in as the experiment is conducted.

When creating data tables, scientists and mathematicians generally place the manipulated variable in the left column and the responding variable in the right column. Since the values for the manipulated variable are chosen by the experimenter before the experiment begins, the first column of the data table can be filled in prior to the experiment. Values of the responding variable can only be filled in as the experiment is conducted.

Data tables are tools for organizing data. In a real laboratory experiment, scientists have to record the data clearly and correctly the first time, during the experiment. It has to be recorded in such a way that not only you can read it, but also your colleagues and even a stranger. A story is told about the Nobel Prize-winner James Watson, who helped discover DNA. His teacher at Indiana University was asked if he suspected that Watson might go on to great things when he graduated. Yes, he answered. Watson would go far. Why? The teacher replied that although Watson did not take great care of his personal appearance, and his desk was a mess, he kept a neat notebook. That is a lesson that even young children can understand.

The data table is also useful in controlling error and identifying patterns. Children can detect blunders when a measurement deviates too much from established patterns (as in the *Marshmallows and Containers* anecdote above), and they can control inevitable measurement error by averaging several trials.

Note that in *The Bouncing Ball* lab, the students performed three trials for each measurement. In other words, to answer the question of how high does the ball bounce when dropped from 40 centimeters, they dropped the ball three times from 40 centimeters and recorded the bounce height, in this case (Figure 4), 27, 22, and 21 centimeters. The students then take the average (they used the median for the average) bounce height for their value of the bounce height. This is done to minimize the effect of measurement and experiential error on the experiment. (See the TIMS Tutors: *Averages* and *Estimation, Accuracy, and Error.*)

Phase 3: Graphing the Data

Graphing is the heart of scientific analysis. Graphs are powerful communication tools that create a picture of the data and "tell its story." They allow you to compare, predict, and infer. If there is a pattern in the relationship between the variables, you are more likely to see it clearly in a graph. Being able to read a graph and produce a graph from data should be a major goal of school science. Graphing cuts across many disciplines: biology, chemistry, sociology, and economics. Students using *Math Trailblazers* work extensively with graphs from kindergarten on.

Bar Graphs vs. Point Graphs

In *Math Trailblazers*, data are mostly graphed as either a bar graph or a point graph; these two types of graphs will thus be the focus of our discussion in this section. Since a graph is a visual representation of the relationship between variables, the type of graph used depends upon the types of variables studied in the experiment.

When both variables are numerical, a point graph is often (though not always) appropriate. In *The Bouncing Ball*, for example, both variables—the drop height and the bounce height—are numerical. Since the values for these variables are numbers and are not discrete—that is, there are values between the data points that make sense, such as 52.5 cm—it is possible to use points and lines on the graph instead of bars to represent the data. Drawing a line or a curve makes sense only when the variables are numerical and there is a pattern in the data. Figure 5 shows a student's graph from *The Bouncing Ball.*

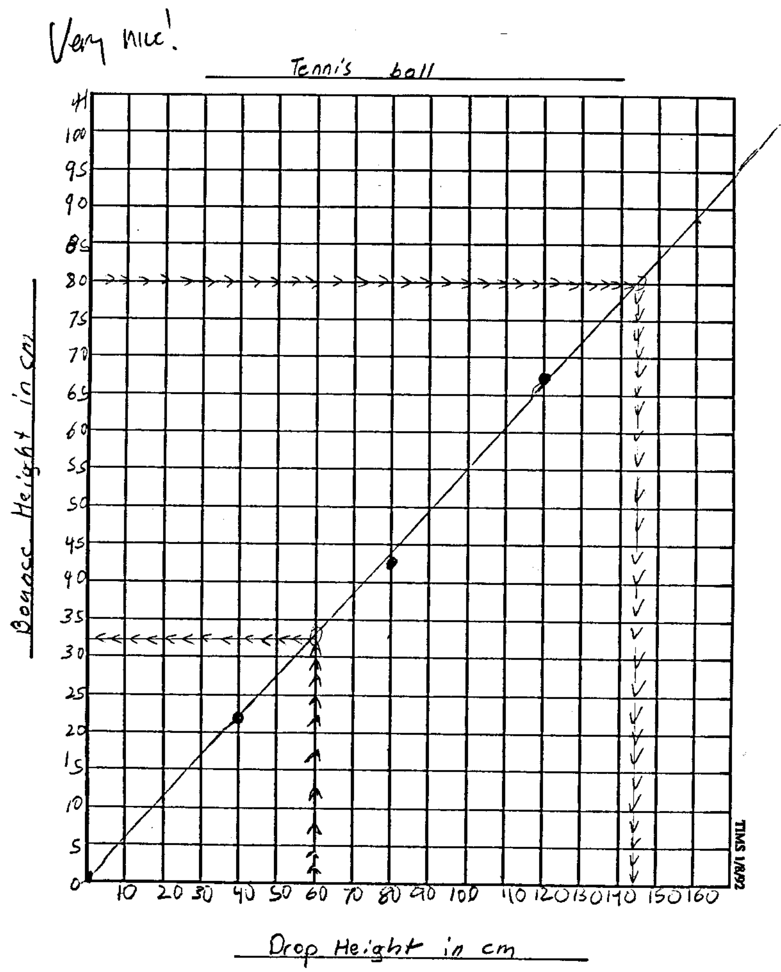

Figure 5: *Point graph from* The Bouncing Ball *showing best-fit lines, interpolation, and extrapolation*

Point graphs are introduced in *Math Trailblazers* in third grade and are used to represent a variety of different mathematical situations as well as to display data from laboratory experiments.

A bar graph is usually best when one of the variables is categorical (qualitative). In *Marshmallows and Containers,* for example, the type of container is a categorical variable and the number of marshmallows is numerical. Figure 6 shows the graph for *Marshmallows and Containers.* There are no values that make sense between the types of containers. Thus it does not make sense to connect them on the graph with a line. A bar graph, therefore, is appropriate for these kinds of situations.

A bar graph is usually best when one of the variables is categorical (qualitative).

Although point graphs are most often used when both experimental variables are numerical, that is not always the case. For example, in the experiment *First Names* from third grade, students collect data about the number of letters in the first names of students in the class. The two primary variables in this experiment are the number of letters (the manipulated variable) and the number of names (the responding variable). A graph for the experiment is shown in Figure 7. While both the variables in this experiment are numerical, they are also discrete—that is, it is not meaningful to speak of $6\frac{1}{2}$ letters in a name or $3\frac{1}{4}$ people who have that number of letters in their names. Thus, we can see why a point graph, in which values between the data points are represented, would not be appropriate in this case. Instead, the data is best represented on a bar graph.

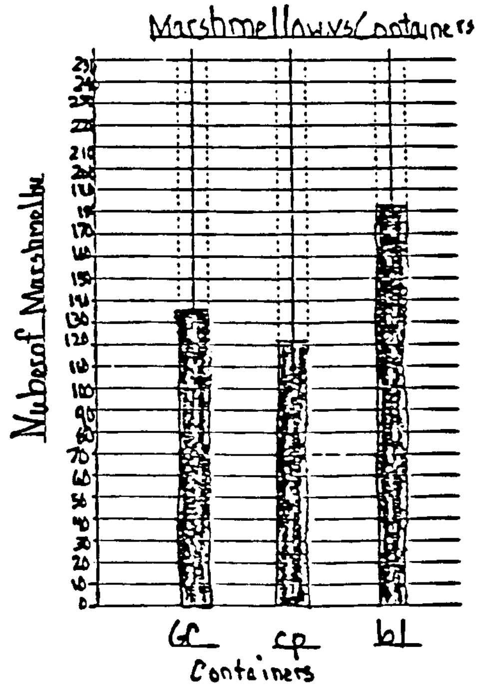

Figure 6: *Bar graph from* Marshmallows and Containers

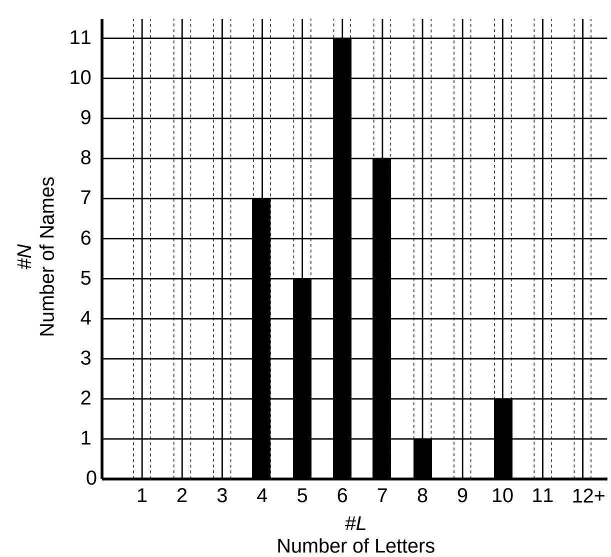

Figure 7: *Graph from* First Names

Labeling the Axes

With most types of data, it is conventional in science to place the manipulated variable along the horizontal axis and the responding variable along the vertical axis. The horizontal axis is labeled with a word or letter describing the manipulated variable and the vertical axis with a word or letter describing the responding variable. In Figure 7, for example, the horizontal axis—the manipulated variable—is labeled *#L,* Number of Letters and the vertical axis—the responding variable—is labeled *#N,* Number of Names.

With most types of data, it is conventional in science to place the manipulated variable along the horizontal axis and the responding variable along the vertical axis.

Once the axes are in place, the children can label the axes with the values for each variable. In the case of the *Marshmallows and Containers* graph in Figure 6, the student wrote in labels to represent the different kinds of containers.

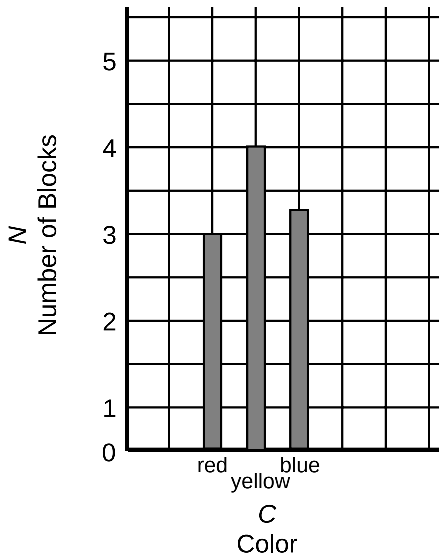

Figure 8: *Common errors in labeling the axes*

A common error in labeling axes is illustrated in Figure 8. This graph shows irregular spacing on both axes. The vertical axis in Figure 8 is incorrect with one space between 0 and 1 and two spaces between the other numbers. Technically, the categorical variables displayed on the horizontal axis do not have to be equally spaced, but it is a good idea to get your students into the habit of spacing their bars across the axis in regularly spaced intervals. This is because when dealing with numerical data, the values *must* be equally spaced.

Scaling the axes for numerical data requires an analysis of the range of the data and comparing it with the number of available intervals on your particular graph paper. Scaling by ones, two, fives, tens, or other numbers might be appropriate depending upon the data for a particular experiment. What is essential, however, is that the intervals are all equal along a given axis. Students will learn that it is best to determine the appropriate interval ahead of time. Otherwise they will end up plotting their initial data points and later discovering that other points will not fit on the graph.

In most cases, the scale on the horizontal axis is independent of the scale on the vertical axis. Students should number the axes in ways that make sense for the data. For example, the horizontal axis in *The Bouncing Ball* graph (Figure 5) is scaled by tens but the vertical axis is scaled by fives. One exception to this is when making scale maps. Here, using different scales for the different axes would create a distorted image and make it difficult to find distances on the map.

As scientists do, students using the TIMS Laboratory Investigations often use their graphs to make predictions about physical phenomena. When making point graphs, therefore, we often encourage students to scale their axes to allow room for extrapolation. (See the section below entitled *Predictions from Point Graphs: Interpolation and Extrapolation* for information about extrapolation.)

Bar Graphs in *Math Trailblazers*

As early as kindergarten, students using *Math Trailblazers* work on graphing concepts, including making and interpreting simple graphs. A quick way to make bar graphs of classroom data is to place self-adhesive notes on a labeled graph. An example of this with *First Names* is illustrated in Figure 9. In this graph, each student placed a self-adhesive note with the data for his or her name on the graph. The data are clearly represented. The one-to-one correspondence between data points and the number of students in the class is particularly apparent in this type of graph, which we use primarily in kindergarten, first, and second grades.

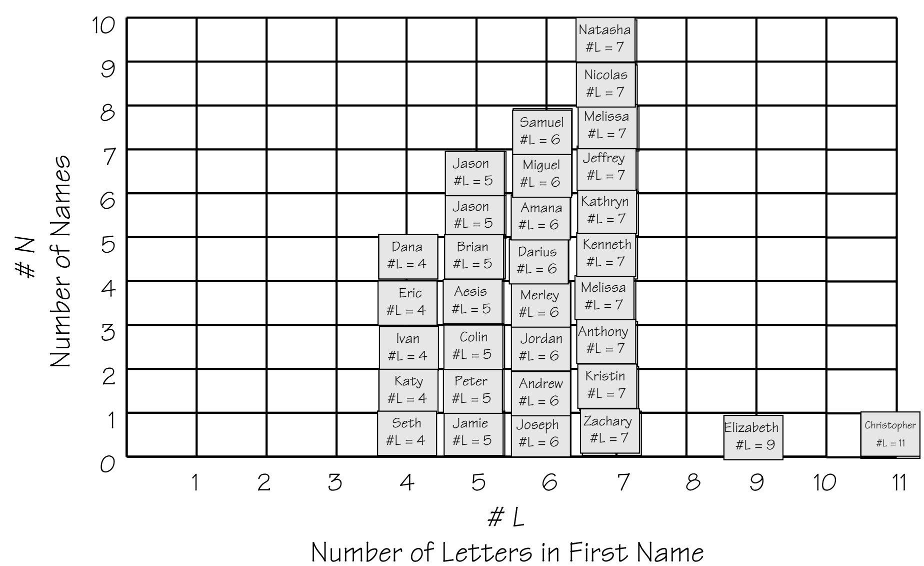

Figure 9: First Names *graph using self-adhesive notes*

In first grade, we introduce a graphing protocol that is different in *Math Trailblazers* than in some other programs. When making bar graphs, we encourage students to create their bars along the vertical lines in the graph rather than in the spaces between the lines. (See Figures 6, 7, and 9.) This does not affect the data or the reading of the graph. Rather it prepares students for making point graphs, where data points are plotted at the intersection of lines extending from the horizontal and vertical axes.

To assist students with this, we have created a special graph paper for making bar graphs. A version of this graph paper is shown in Figure 10. The dark vertical grid lines across the page are where the data for the manipulated variable is plotted. These lines are surrounded on both sides by a pair of dashed guide lines. Students make bars by coloring in the space on either side of the dark vertical lines. The result is a straight bar that is centered along a vertical line. The values for the manipulated variable are indicated on the horizontal axis directly below each bar.

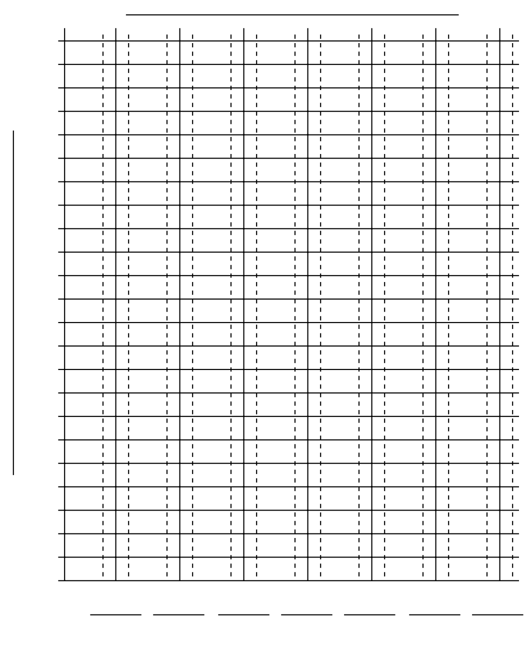

Figure 10: *TIMS bar graph paper*

The benefit of making bar graphs in this manner is most apparent in third grade as students make the transition to creating point graphs. To simplify students' initial attempt at creating a point graph, they first graph a data set as a bar graph and then convert the bar graph to a point graph. This is shown in Figure 11.

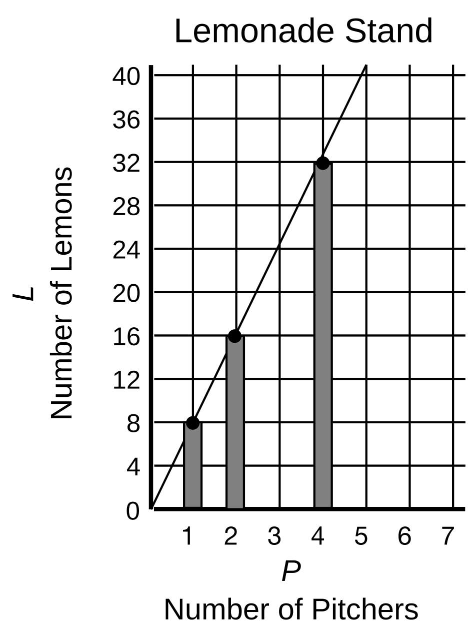

Figure 11: *Transition from bar graphs to point graphs in third grade*

Point Graphs: Fitting Lines and Curves

With point graphs, the data are plotted at the junction of the values of each variable. Once the data are plotted, we look for patterns. If the data points form a line or close to a line, we try to fit a line through the points. If the data form a curve, we try to fit a smooth curve through the points.

Figure 11 shows a graph in which the data points lie exactly on a straight line. Fitting a line to these data points is simply a matter of laying down a ruler and connecting the points.

For most experiments, we cannot expect the data to be so precise. In these experiments, the data may lie close to, but not exactly on a straight line. The "zigzags" in the data are due to experimental error. (See the TIMS Tutor: *Estimation, Accuracy, and Error.*) To average the error, one fits a line which comes as close to the data points as possible even though the line may not pass through any of these points. To assure a good fit, you would like as many points above the curve as below. You do not want to force the line through two points while missing the third by a mile. It is better to miss them all but come close to all than be too far from any one point.

As shown on *The Bouncing Ball* graph in Figure 5, a line can be fit to the data points, not by using some complicated statistical procedure, but simply "by eye." The student uses a clear ruler or a thread and moves it around until it fits the data points as closely as possible. This best-fit line is useful for minimizing error and making predictions.

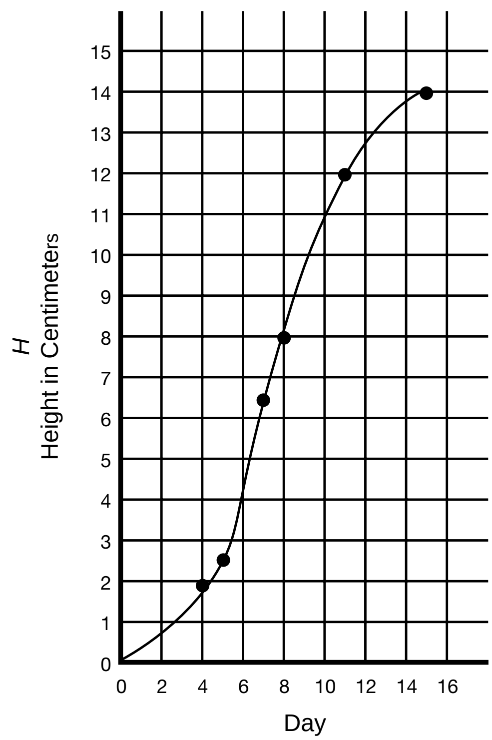

Figure 12: *Graph from* Plant Growth

Of course, not all experimental relationships result in data that yields straight lines on graphs. In *Math Trailblazers*, we also explore data that exhibits other kinds of patterns. Figure 12 shows the graph from a fourth-grade experiment, *Plant Growth,* where the data shows a pattern, but not a straight line.

When you can fit a smooth curve through the data points, you have uncovered one of nature's secrets—that the variables are related, that there is a predictable pattern to the data. We are then not restricted only to our specific data points, but can now use the pattern to predict what the value of the responding variable will be for any value of the manipulated variable.

Predictions from Point Graphs: Interpolation and Extrapolation

A major goal of mathematics and science is to find patterns in data and to use the patterns to make predictions. Interpolating and extrapolating on a point graph are two ways to do this.

A major goal of mathematics and science is to find patterns in data and to use the patterns to make predictions. Interpolating and extrapolating on a point graph are two ways to do this.

Using the graph to find data points that lie between those in your data table is called **interpolation.** Inter means between or among. The simplest situation is when the pattern of data points produces a straight line. In the graph in Figure 5, the student interpolated to predict that a ball dropped from 60 cm would bounce about 32 cm. Note that in Figure 5 the student showed how she made her interpolation by starting at 60 cm on the horizontal axis, drawing a line (with arrows) up to the best-fit line, and then drawing a horizontal line to find the corresponding value of the bounce height—about 32 cm.

Extrapolation is an attempt to predict information beyond the last data point. *Extra* means outside or beyond. To extrapolate, we must extend the line into a region where there is no data. You can do this easily by laying a ruler on the straight line, extending the line, and reading off your prediction. In Figure 5, the student extrapolated to predict that a ball that bounced 80 cm was dropped from a height of about 144 cm. Here, the student began with values on the vertical axis (the bounce height) and predicted a value on the horizontal axis (the drop height). It is possible to interpolate and extrapolate in either of two directions: from values on the horizontal axis to values on the vertical axis or vice versa.

Having used these techniques to make predictions, it is important to have the children check their predictions experimentally and see how close their new data comes to the curve. This is one of the joys of science—to see that nature is often regular and predictable and that you can make predictions that come true! Checking predictions, though time-consuming, is worthwhile because it reinforces connections between mathematical abstractions and the real world.

Phase 4: Analyzing the Data

After making the graph, students have explored the relationship between the variables in four ways: with the physical materials, in the picture, in the data table, and in the graph. The last phase of the TIMS Laboratory Method is the analysis of the entire situation, where students explore the relationship quantitatively and represent it symbolically (i.e., with numbers).

One way to structure this analysis is to ask a series of questions. The questions usually begin on the literal level: *Did the tallest container hold the most marshmallows? How high did the tennis ball bounce when dropped from 60 cm?* More demanding questions require prediction: *How many marshmallows would two bowls hold? If a ball bounced to 45 cm, what height was it dropped from?* Asking what would happen if one of the fixed variables is changed can build a broader understanding of the situation: *What would happen if we used large instead of miniature marshmallows? What would happen if we used a Super ball instead of a tennis ball?* This quantitative analysis of the data is one place in *Math Trailblazers* where students regularly practice and reinforce arithmetic and other math skills.

The end of the investigation may be a completely satisfying answer to the original question, but, more often than not, the end is another question that can lead to further investigations. *Marshmallows and Containers,* for example, might lead to an investigation of the liquid capacities of other short and tall containers. After an initial experiment with tennis balls in *The Bouncing Ball,* children carry out an investigation using another type of ball and compare the results from the two experiments.

Picture, Table, Graph, and Questions: Putting It Together

Each of the four phases described above may require one or more class periods. In addition, time may be spent becoming familiar with the equipment at the beginning and on further experiments at the end. Thus, a lab is an extended activity that may last a week or even longer. This is much longer than a typical mathematics or science lesson, but there are significant benefits.

Identifying variables, drawing pictures, measuring, organizing data in tables, graphing data, and looking for patterns are part of many scientists' work. Students are thus inducted via this method into the authentic practice of science.

First, the four phases simplify the scientific method enough for children to use, but not so much that it fails to resemble what scientists do. Identifying variables, drawing pictures, measuring, organizing data in tables, graphing data, and looking for patterns are part of many scientists' work. Students are thus inducted via this method into the authentic practice of science.

The method fosters children's sense-making. Children handle numbers they have generated themselves by counting or measuring, numbers that are thus meaningful to them. As they deal with experimental error, they develop number sense and estimation skills. As they look for patterns in their tables and graphs, they make sense of the numbers before them. Arithmetic in context is more understandable.

The approach is multimodal, which has benefits for both individual students and heterogeneous groups of students. The multiple representations of relationships between the variables permit problems to be solved in more than one way, allowing different students to approach the same content in ways they understand. The container that holds the most marshmallows, for example, can be found from the graph, from the data table, or from the marshmallows themselves. A prediction about a bounce height might be obtained by extrapolating on the graph or by extending patterns in the data table, and can then be verified using the apparatus. Students can compare these various approaches, thus helping them make connections within mathematics as well as between the informal mathematics of their everyday experience and more formal mathematics.

Mathematics in Context

Two principles underlie the TIMS Laboratory Method. First, an investigation should begin within the children's own experiences. Children use objects from their everyday lives to investigate a familiar situation. Children's everyday knowledge, like a scientist's theory, provides a framework for interpreting the results of the investigation. Without that framework, the investigation would remain hollow and meaningless.

The second principle is that an investigation should also transcend children's everyday experiences. The exploration must go somewhere; it must lead the children both to a better understanding of the immediate situation and to improved skills, understandings, habits, and attitudes. The concepts can then be extended and transferred to new contexts.

Balancing these principles requires teacher judgment. The key is to enable students to follow their own ideas, but with the intention that those ideas will lead somewhere. How much scaffolding to provide, how much to guide students in directions that are fruitful rather than sterile, must be decided by the teacher in context. The goal is that students should advance not only in skill and understanding, but also in autonomy and perseverance. Just how much structure to provide along the way is perhaps a teacher's most important and difficult job.

The TIMS Laboratory Method helps children connect their everyday experiences with formal mathematics. As they investigate everyday situations quantitatively, children handle variables, explore relationships between variables, master a few powerful techniques for representing these relationships, and use these multiple representations to generate a wide variety of problem solutions. By beginning and ending in familiar situations, the abstractions of mathematics are linked to children's everyday knowledge. As students master this method, they become increasingly autonomous and flexible in its application. Then we can truly say they understand the fundamentals of *doing* science.

References

Archambault, Reginald D. (ed.), *John Dewey on Education: Selected Writings.* Modern Library, 1964.

Bruner, Jerome S. "The Course of Cognitive Growth." *American Psychologist* 19(1), pp. 1–15, 1964.

Dewey, John. "Science as Subject-Matter and as Method." *Science* 31(787), pp. 121–7, January 28, 1910.

Goldberg, Howard, and F. David Boulanger. "Science for Elementary School Teachers: A Quantitative Approach." *American Journal of Physics* 49 (2), pp. 120–124, 1981.

Goldberg, Howard, and Philip Wagreich. "Focus on Integrating Science and Math." *Science and Children* 2(5), pp. 22–24, 1989.

Goldberg, Howard, and Philip Wagreich. "A Model Integrated Mathematics and Science Program for the Elementary School." *International Journal of Educational Research* 14(2), pp. 193–214, 1990.

Hiebert, James. "A Theory of Developing Competence with Written Mathematical Symbols." *Educational Studies in Mathematics* 19, pp. 333–355, 1988.

Isaacs, Andrew C., and Catherine Randall Kelso. "Pictures, Tables, Graphs, and Questions: Statistical Processes." *Teaching Children Mathematics* 2 (6), pp. 340–345, 1996.

Isaacs, Andrew C., Philip Wagreich, and Martin Gartzman. "The Quest for Integration: School Mathematics and Science." *American Journal of Education* 106 (1), pp. 179–206, 1997.

Lesh, Richard, Thomas Post, and Merlyn Behr. "Representations and Translations among Representation in Mathematics Learning and Problem Solving." In C. Janvier (ed.), *Problems of Representation in the Teaching and Learning of Mathematics.* Lawrence Erlbaum Associates, Hillsdale, NJ, 1987.

National Research Council. *National Science Education Standards.* National Academy Press, Washington, DC, 1996.

Silver, Edward. "Using Conceptual and Procedural Knowledge: A Focus on Relationships." In J. Hiebert (ed.), *Conceptual and Procedural Knowledge: The Case of Mathematics.* Lawrence Erlbaum Associates, Hillsdale, NJ, 1986.